12.333 Problem Set 3: Vorticity and Rossby waves (Due: 17 Apr 2014)

(4 questions)

1. Consider barotropic flow ($L_R = \infty$). An axisymmetric vortex patch on an f-plane (Coriolis parameter $f = f_0$ = constant; $\beta = 0$) has absolute vorticity

\[ \zeta_a = \begin{cases} f_0 + Z , & r < R \\ f_0 , & r > R \end{cases} \]

where $Z$ is a constant relative vorticity. Calculate the profile of azimuthal velocity as a function of $r$.

2. Consider the group velocity of Rossby waves on a resting shallow water fluid of depth $D$ on a $\beta$-plane ($f = f_0 + \beta y$). The component of wavenumber in the latitudinal direction is $l = L_R^{-1}$, where $L_R$ is the deformation radius. Compare the maximum eastward component of group velocity and the maximum westward component; what is the zonal wavenumber of these waves?

3. Consider now the group velocity of stationary ($c = 0$) Rossby waves on a shallow water fluid of depth $D$ on a $\beta$-plane ($f = f_0 + \beta y$) with constant westerly flow $u_0$. Calculate the maximum eastward component of group velocity; what are the zonal and meridional wavenumbers of these waves?

4. Consider stationary ($c = 0$), vertically propagating, Rossby waves on a midlatitude $\beta$-plane at $45^\circ$N with uniform westerly background flow $U$ and with uniform $N^2 = 4 \times 10^{-4} \text{s}^{-2}$; the appropriate scale height is $H = 7\text{km}$. For waves with latitudinal wavenumber $l = \pi/ (5000 \text{km})$, plot the Rossby critical velocity $U_c$ (such that propagation occurs only for $0 < U < U_c$) as a function of $s$, where $s$ is the number of wavelengths around a latitude circle. Discuss the implications of your results.